Warmup (Lesson 10)

1. Find $\frac{dy}{dx}\Big|_{x=4}$ if $y = 3e^x/\sqrt{x}$

$= 3e^x x^{-1/2}$

$\frac{dy}{dx} = 3e^x x^{-1/2} + 3e^x\left(\frac{-1}{2}x^{-3/2}\right)$

$\frac{dy}{dx}\Big|_{x=4} = 3e^4(4)^{-1/2} + 3e^4\left(\frac{-1}{2}\right)(4^{-3/2})$

$= 3e^4\left(\frac{1}{\sqrt{4}}\right) + 3e^4\left(\frac{-1}{2}\right)\left(\frac{1}{\sqrt{4}^3}\right)$

$= 3e^4\left(\frac{1}{2}\right) + 3e^4\left(\frac{-1}{2}\right)\left(\frac{1}{2^3}\right)$

$= \frac{3}{2}e^4 - \frac{3}{16}e^4$

$= \left(\frac{3}{2} - \frac{3}{16}\right)e^4$

$= \left(\frac{24}{16} - \frac{3}{16}\right)e^4$

$= \boxed{\frac{21}{16}e^4}$

2. What is the derivative of $f(x) = \frac{1}{x}$?

$= x^{-1}$

$f'(x) = -x^{-2}$

$= \boxed{-\frac{1}{x^2}}$

Lesson 10: Quotient Rule and Trig Derivatives

QUOTIENT RULE: $\boxed{\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}}$

Ex 1 $y = \frac{1}{x}$ $\quad \frac{d}{dx}[1] = 0 \quad \frac{d}{dx}[x] = 1$

$y' = \frac{x(0)^{0} - 1(1)}{x^2} = \boxed{\frac{-1}{x^2}}$ $\quad * \boxed{\sin^2 x + \cos^2 x = 1} *$

Ex 2 $y = \frac{\sin x}{\cos x} \quad \frac{d}{dx}[\sin x] = \cos x \quad \frac{d}{dx}[\cos x] = -\sin x$

$y' = \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2} = \frac{\cancel{\cos^2 x + \sin^2 x}^{1}}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$

Ex 3 $y = \frac{1}{\cos x} \quad \frac{d}{dx}[1] = 0 \quad \frac{d}{dx}[\cos x] = -\sin x$

$y' = \frac{\cos x(0) - 1(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos^2 x}$

$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$= \boxed{\sec x \tan x}$

Ex 4 $y = \frac{1}{\sin x}$ $(= \csc x)$

$\rightarrow y' = -\csc x \cot x$

Ex 5 $y = \frac{\cos x}{\sin x}$ $(= \cot x)$

$\rightarrow y' = -\csc^2 x$

$f(x)$	$f'(x)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

MEMORIZE THESE!

Ex 6 $y = \dfrac{\pi - \sin x}{x^2}$ $\quad \frac{d}{dx}[\pi - \sin x] = -\cos x$ $\quad \frac{d}{dx}[x^2] = 2x$

$$y' = \frac{x^2(-\cos x) - (\pi - \sin x)(2x)}{(x^2)^2}$$

$(x^2)^2 \leftarrow x^{2\cdot 2} = x^4$

Ex 7 $f(x) = \dfrac{7a}{x^2 - 1}$ where a is a constant

$\frac{d}{dx}[7a] = 0$ ($7a$ is a constant) $\quad \frac{d}{dx}[x^2 - 1] = 2x$

$$f'(x) = \frac{(x^2-1)(0) - 7a(2x)}{(x^2-1)^2} = \boxed{\frac{-14ax}{(x^2-1)^2}}$$

Ex 8 $y = x \cdot \sec x$ $\quad \frac{d}{dx}[x] = 1$ $\quad \frac{d}{dx}[\sec x] = \sec x \tan x$

$$y' = 1(\sec x) + x(\sec x \tan x)$$

$$= \boxed{\sec x + x \sec x \tan x \text{ OR } \sec x(1 + x \tan x)}$$